220

Study Material for Students Appearing in Compartment Examination-2005

317 | Gift

(11)

Mathematics

SCERT

स्वाध्यायान्मा प्रमदः

State Council of Educational Research and Training
Varun Marg, Defence Colony, New Delhi-110024

317/Gift

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1. General Instructions to Students

1. Keep your mind cool and be stress free at the start of the examination.
2. Read all the questions carefully and mark the questions which you think you can solve without any problem.
3. State your answer with the question for which you are very much confident that you can solve it correctly without any cuttings. The question which you can attempt very well can be in any section. e.g. suppose you read the question paper and find that you can answer Q. No.-16 which is in section B, very well, then without any hesitation, start your answer with Q. 16 Don't worry your marks will not be deducted but this step will be in your favour, because you will gain confidence and solve the rest of the paper without any stress.
4. After finishing the answer of one question, mark a small line so that examiner can differentiate two answers easily and quickly.
5. Always write correct question number. For example. If you are answering in the beginning Q.No.21 from Section C then you should write Q. No. 21 and then give answer. Some students write Q.No.-1 instead of Q.No.21, so for this reason after attempting the question correctly they loose marks.
6. Do the rough work of the question in front of it only so that while checking, the examiner will look into it and if you have done some mistake in writing the answer from your rough work to fair work, the examiner will check your rough work and won't deduct your marks.
7. Try to attempt all questions. It you don't know the answer, just write the given instruction in the question. If the question demands the figure, draw it. Write the formulae/rule required to solve the question so that you can earn some marks from that question.
8. Solve all the solved examples of your NCERT Book 4 to 5 times before your paper and go through all the theorems thoroughly.
9. Always revise your answer in the end so that if you have done some mistake in solving the question or writing any step/answer, you can correct it while revision and save your marks.
10. Most Important

    Divide your Three hours as following:
    1. 15-20 minutes to read the question paper.
    2. For Section A; 3 marks question per question devote only 5 to 6 min. If you are unable to get the answer just leave it and attempt next question.
    3. For section B, 4 marks question per question devote only 6 to 7 min.
    4. For Section C, 6 marks question per question devote only 8 to 10 minutes.
    5. Keep 15-20 minutes for revising the answers you have done in the answer book.
11. Tie all the extra sheets tightly, if you have taken 10 minutes before the time expires.
12. A survey was conducted in 2 to 3 Schools where remedial classes of compartment students are going on in the summer vacations We did the survey of those students who got compartment in Mathematics. After talking to the students for some time, we came to the conclusion that students preferred to study and revise the following topics which they felt were easy to understand.

Sr. No.	Topics	Marks
1.	Linear Equations	7
2.	HCF & LCM	3
3.	Rational Expressions	3
4.	Arithmetic Progression	6
5.	Installments	6
6.	Income Tax	6
7.	Statistics	12
8.	Menstruation	10
9.	Construction	4
10.	Quadratic Equation	7
	Total Marks	**64**

If the students revise these chapters thoroughly, they can get sufficient marks to pass their compartment exams.

2. CLASS-X PAPER-I: SOLUTIONS

Candidates must write the code on the title page of the answer book.

- Please check that this question paper contains 12 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 25 questions.
- **Please write down the Serial Number of the question before attempting it.**

MATHEMATICS

[Time Allowed: 3 Hours] *[Maximum marks: 100]*

General Instructions:

(i) *All questions are compulsory.*

(ii) *The question paper consists of 25 questions divided into three sections —**A, B and C**. Section **A** contains **10** questions of 3 marks each, Section **B** is of **10** questions of **4** marks each and section **C** is of **5** questions of **6** marks each.*

(iii) *There is no overall choice. However, an internal choice has been provided in two questions of three marks each, two question of four marks each and two questions of six marks each.*

(iv) *In question on construction, the drawing should be neat and exactly as per the given measurements.*

(v) *Use of calculators is not permitted.*

SECTION-A

Question numbers 1 to 10 Carry 3 marks each

1. Express the following as a rational expression in lowest terms:

$$\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1} \div \frac{x^2-2x+4}{x^2+2x-3}$$

Answer. *See chapter 3 Rational Expression Page No. 58-59*

Please see example 26 P.No. 58 in N.C.E.R.T. Book.

(a) $x^3-8 = x^3-2^3$

x^3- 8 can be writeen as x^3- 2^3; this is of the type

a^3-b^3 and $a^3-b^3 = (a-b)(a^2+ab+b^2)$

$x^3-8 = x^3-2^3 = (x-2)(x^2+2x+4)$

(b) x^2-4 can be written as x^2-2^2; this is of the type

a^2-b^2 and $a^2-b^2 = (a-b)(a+b)$

$\therefore\ x^2-4 = (x-4)(x+4)$

(c) x^2+6x+8 factorize this polynomial by splitting of middle term.

$x^2 + 6x + 8 = x^2+(4+2)x+8$ ($\therefore$ $4+2 = 6$ and $4\times2=8$)

$= x^2+4x+2x+8$

$= (x^2+4x)+ (2x+8)$

Taking x from 1st bracket common and 3 from 2nd bracket common.

$x(x+4) + 2(x+4) = (x+4)(x+2)$ *(taking x+4 as common bracket we get)*

(d) x^2-2x+1 factorize this polynomial by spliting of middle term.

$x^2-2x+1 = x^2-(1+1)\,x-11 \qquad (\therefore 1+1=2$
$1\times 1=1)$

$= x^2-x-x-1$
$= (x2-x) + (-x+1)$
Taking x common form 1st bracket and -1 from 2nd bracket, we get
$x(x-1) - 1(x-1)$
Taking $(x-1)$ common bracket we get
$(x-1)\,(x-1)$

(e) x^2+2x+4 Here middle term cannot be split up so it cannot be factorized . Hence leave this polynomial as it is.

(f) x^2+2x-3 factorize it by spitting the middle term.

$= x^2+(3-1)x-3 \qquad \left[\begin{array}{l}\therefore 3-1=2\\ 3x(-1)=-3\end{array}\right]$

$= x^2+3x-x-3$
$= (x^2+3x) + (-x-3)$
Taking x common from 1st bracket and -1 from 2nd bracket we get
$x(x+3)-1\,(x+3)$
$(x+3)\,(x-1)$
from a, b, c, d, e, & f

$$\frac{(x-2)(x^2+2x+4)}{(x-2)\,(x+2)} \times \frac{(x+4)\,(x+2)}{(x-1)(x-1)} \div \frac{x^2+2x+4}{(x-1)(x+3)}$$

Replace (division sign) by × (multipilcation sign) and taking reciprocal of $\dfrac{x2+2x+4}{(x-1)\,(x+3)}$

$$\frac{\cancel{(x-2)}\cancel{(x^2+2x+4)}}{\cancel{(x-2)}\,\cancel{(x+2)}} \times \frac{(x+4)\,\cancel{(x+2)}}{(x-1)\cancel{(x-1)}} \times \frac{\cancel{(x-1)}\,(x+3)}{\cancel{(x^2+2x+4)}}$$

$$\frac{(x+4)(x+3)}{(x-1)} = \frac{x^2+(4+3)x+12}{x-1} = \frac{x^2-7x+12}{x-1}$$

$$Ans \quad \frac{x^2+7x+12}{x-1}$$

2. Find 10th term from end of the A.P, 914, ..., 254

Answer:

Given A.P. = 4,9,14, ..., 254
This question is from chapter 5 Arithematic Progressions. *Ex. 5.1 P.No. 89, Q.-4*
Here a= 4, d= a_2-a = 9-4=5, t_n= 254
We know that $t_n = a+(n-1)d$

$$254 = 4 + (n-1)5$$
$$254 = 4+5n-5$$
$$254 = 5n-1$$
$$254+1 = 5n$$
$$255 = 5n$$
$$\frac{255}{5} = 51$$
$$n = 51$$

∴ The given A.P. has 51 terms.

The 10th term from the end is 42nd term

∴ 10th terms from the end t_{42}

$t_{42} = a+(42-1)d$

$t_{42} = 4+41\times5$

$= 4+205 = 209$

Alternative Method

Rewriting the A.P. in the reverse order, we have 254, 249, 244, ...14, 9, 4.

Here a = 254, d= 249-254= -5

we will find the 10th term of this A.P.

$t_{10} = a+(n-1)d$

$t_{10} = 254+(10-1)(-5)$

$= 254+9(-5)$

$= 254-45$

$= 209$

$\therefore$ 10th term from the end of the given A.P.

4,9,14, ... is 209.

3. Solve the following system of linear equation:

$$ax+by = a-b$$

$$bx- ay = a+b$$

Answer: $ax+by = a-b$ ______________________ (1)

$bx- ay = a+b$ ______________________ (2)

See Chapter 1 linear Eq^n. Ex 1.2 P.No. 13

Applying elimination method (by equating Cofficients) Multiplying eq^n(1) by (b) and eq^n (2) by (a) we get

$[ax+ by = a-b\,]\times b$

$[bx-ay = a+b]\times a$

$\cancel{ab}x + b^2y = ab-b^2$

$-\cancel{ab}x - a^2y= a2 +ab$

$\quad + \quad - \quad -$

$(a^2 + b^2)y = \cancel{ab}-b^2 -a^2-\cancel{ab}$

$y\,(a^2+b^2) = -(a^2+b^2)$

$$y = \frac{-(a^2+b^2)}{(a^2+b^2)} = -1$$

$y = -1$

Putting value of y in eq^n 1 we get,

$ax+b(-1) = a-b$

$ax -b = a-b$

$ax= a-\cancel{b}+\cancel{b}$

$ax = a$

$$x= \frac{a}{a} = 1$$

Hence Answer $x=1,\ y = -1$

4. Find the L.C.M. of the following polynomials:

$2x^3 - 128;\ x - 9x + 20\ ;\ x^2-16$

Answer:

L.C.M. of $2x^3 - 128;\ x^2 - 9x + 20\ ;\ x^2-16$

See Chapter 2 H.C.F. & L.C.M. of polynomials. Ex. 2.2, P.No. 38, Q. 1, 2, 3.

Let $p(x) = 2x^3-128, \quad q(x) = x^2-9x+20, r(x) = x^2-16$

(a) $p(x) = 2x^3-128$

$= 2(x^3-64)$ *Taking 2 Common*

$= 2(x^3-4\times4\times4) \qquad = 2\ (x^3-4^3)$

$= 2(x-4)\ (x+4x+16) \qquad$ ($x3-4^3$ *is of the type* a^3-b^3)

$a^3-b^3= (a-b)\ (a^2+ab+b^2)$

(b) $q(x) = x^2-9x+20$

$= x^2-(5+4)x+20 \qquad$ *(By splithing up middle term* $5+4=9,$ *)*

$5\times4=20$

$= x^2-5x-4x+20$

$(x^2-5x)+\ (-4x+20)$

Taking x common from 1st bracket and -4 common from 2nd bracket, we get,

$x(x-5)\ -4(x-5)$

Taking (x-5) common from this we get.

$(x-5)(x-4)$

(c) $r(x) = x^2-16$

$= x^2-4^2 \qquad (\therefore 16= 4\times4 = 4^2)$

$= (x-4)\ (x+4) \qquad (\therefore a^2-b^2 = (a-b)\ (a+b))$

L.C.M. of p(x), 9(x) r(x) from a,b,c, we get.

$2(x-4)\ (x-5)\ (x+4)\ (x^2+4^x+16)$

Answer $2(x-4)\ (x+4)\ (x-5)\ (x^2+4x+16)$

5. **Solve for *x*:**

$$\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x};\ a\neq 0,\ b\neq 0,\ x\neq 0$$

Or

Solve for x:

$a\ bx^2 + (b^2-ac)\ x-bc=0$

Answer

$$\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x}$$

see chapter 4 quadratic Eq^n, Ex 4.2 P.No 67.

Taking x on one side, we get,

$$\frac{1}{a+b+x}-\frac{1}{x} = \frac{1}{a}+\frac{1}{b}$$

Take L.C.M. on both sides we get,

$$\frac{x-(a+b+x)}{(a+b+x)x} = \frac{b+a}{ab}$$

$$\frac{\cancel{x}-a-b-\cancel{x}}{(a+b+x)x} = \frac{b+a}{ab}$$

$$\frac{-(a+b)}{(a+b+x)x} = \frac{(a+b)}{ab}$$

$$\frac{-1}{x\,(a+b+x)} = \frac{1}{ab}$$

by cross multiplying we get,

$-ab = x\,(a+b+x)$

$-ab = ax + bx + x^2$

$ax+bx+x^2+ab=0$

$x^2 + (a+b)\,x+ab=0$

$x^2 + ax + bx + ab=0$

$(x^2 + ax) + (bx+ab)=0$

$x(a+x) + b(x+a) = 0$

$(x+a)\,(x+b) = 0$

$x+a = 0$ or $x+a = 0$

$x = -a$ or $x=-b$

Answer: $x= -a, -b$

Or

$abx^2 + (b^2-ac)x-bc=0$

$abx^2 + b^2x-acx -bc=0$

$(abx^2+b^2x)- (acx-bc)=0$

$bx(ax+b) -c\,(ax+b)=0$

$(ax+b)\,(bx-c)=0$

$ax+b=0$ or $bx-c=0$

$ax=-b$ or $bx=c$

$x= \frac{-b}{a}$ or $x= \frac{c}{b}$

Answer: $x = \frac{-b}{a}, \frac{c}{b}$

6. Find the number of terms of the A.P. 54, 51, 48,..... so that their sum is 513.

Or

If the *nth* term of an A.P. is (2*n* +*1*), find the sum of first *n* terms of the A.P.

See Chapter No.5 Arithematic Progression Ex.5.1 P.No 90 From Q. No. 15-19

Answer:

The given A.P. is

54, 51, 48,....................................

Here $a= 54$, $d= a_2-a_1 = 51- 54= -3$

$Sn = 513$ (sum of *n*th term)

$$Sn = \frac{n}{2}\,(2a+(n-1)d)$$

$$513 = \frac{n}{2}\,(2\times 54+(n-1)3)$$

$513 \times 2 = n\,(108+(3n)+3)$

$1026=n\,(108+3-3n)$

$1026 =111n-3n_2$

$3n^2-111n+1026=0$

Taking 3 common from the above quadratic Eq^n

$3(n^2-37n+342)=0$

$n^2-37n + 342 = 0 \qquad (\because n^2-3n+342=\frac{0}{3}=0)$

$$n^2-(18+19)n+342=0 \qquad \left\{\begin{matrix} \because 18+19=37 \\ 18\times 19=342 \end{matrix}\right\}$$

$$n^2-18n-19n+342=0$$

$$n-(n-18)-19(n-18)=0$$

$$(n-18)(n-19)=0$$

$$n-18=0 \quad or \quad n-19=0$$

$$n=18 \quad or \quad n=19$$

$\therefore$ *Required number of term in given A.P. is 18 or 19.*

Or

Answer: $t_n=2n+1$

Let $n=1$ *then*

$$t_1=2\times(1)=2+1=3$$

for $n=2 \quad t_2=2\times2+1=4+1=5$

$n=3 \quad t_3=2\times3+1=6+1=7$

$n=4 \quad t_4=2\times4+1=8+1=9$

$\therefore$ *the given A.P.is*

$$3,5,7,9..................2n+1$$

Here $a=3,\ d=5-3=2$

Sum of first n terms $=Sn=\frac{n}{2}(2a+(n-1)d)$

$$=\frac{n}{2}(2\times3+(n-1)\times2)$$

$$=\frac{n}{2}(6+2n-2)$$

$$=\frac{n}{2}(4+2n)$$

$$=\frac{n}{\cancel{2}}\times\cancel{2}(2+n)$$

$$=n(n+2)$$

$$=n^2+2n$$

Hence the sum of first n terms of given A.P. is n^2+2n

7. A loan of Rs. 10,815 is to be returned in three equal half-yearly instalment. Calculate the amount of each instalment, if the rate of interest is $13\frac{1}{3}$% per annum, compounded half-yearly .

Answer: See chapter No.6 Instalment Ex 6.2 P.No 104 and Example No. 7, P.No 102

loan Amount= Rs 10,815/-

No of instalment =3

rate of intt. $=13\frac{1}{3}\%$ *P.a*

$$=\frac{40}{3}\%\ P.a$$

But rate of intt. is calculated half yearly

$$r=\frac{{}^{20}\cancel{40}}{3\%}\times\frac{1}{\cancel{2}}=\frac{20}{3}\%$$

Let amount of instalment = Rs x/- Let P_1, P_2 *and* P_3 *be the principal of first, second, and third instalment respectively*

$$Then\ P1=\frac{x}{1+\frac{\cancel{20}}{\cancel{200}}}=\frac{x}{\frac{1+1}{15}}=\frac{x}{\frac{16}{15}}$$

$$=\frac{15}{16}x$$

$$\text{Then } P2 = \frac{x}{\left(\frac{1+\cancel{20}}{\underset{15}{\cancel{300}}}\right)^{2}} = \frac{x}{\left(\frac{16}{15}\right)^{2}}$$

$$= \left(\frac{15}{16}\right)^{2} x$$

$$P_3 = \frac{x}{\left(\frac{1+\cancel{20}}{\underset{15}{\cancel{300}}}\right)^{3}} = \frac{x}{\left(\frac{16}{15}\right)^{3}}$$

$$= \left(\frac{15}{16}\right)^{3} x$$

$$\left[\begin{array}{l} A=P(1\times\frac{r}{100})^{n} \\ \text{Here } A= \text{Rs } x \\ P= ?\ n=1 \\ r=\frac{20}{3}\% \\ x=p(1+\frac{20}{3\times100})^{1} \\ p=\frac{x}{1+\frac{20}{300}} \end{array}\right]$$

Given $P_1 + P_2 + P_3 = \text{Rs } 10,815$

or

$$\frac{15}{16}x + \left(\frac{15}{16}\right)^{2}x + \left(\frac{15}{16}\right)^{3}x = 10,815$$

$$\frac{15}{16}x\left(1+\frac{15}{16}+\left(\frac{15}{16}\right)^{2}\right) = 10.815$$

$$\frac{15}{16}x\left(\frac{256+15\times16+225}{256}\right) = 10,815 \qquad \left(\because \begin{array}{l}15^{2}=225\\16^{2}=256\end{array}\right)$$

and by taking L.C.M

$$\frac{5}{16}x\left(\frac{256+240+225}{256}\right) = 10,815$$

$$\frac{15}{16}x\left(\frac{721}{256}\right) = 10,815$$

$$\frac{15}{16}x = \left(\frac{10815+256}{721}\right)$$

$$x = \frac{\overset{\cancel{721}\ \cancel{2163}}{10815\times256x16}}{\underset{1}{\cancel{721}}\times\underset{1}{\cancel{15}_{\cancel{3}}}} = 256\times16 = \text{Rs } 4096$$

Hence amount of each instalment is Rs 4096.

8. A fan is available for Rs. 970 cash or Rs. 210 as cash down payment followed by three equal monthly instalment of Rs. 260 each. Find the rate of interest charged under instalment plan.

Answer: Cash Price of fan= Rs 970/-

Cash down payment = Rs 210/-

Amount of each instalment= Rs 260/-

(See chapter 6 instalment Ex 6.1 P.No 99 and Example 2 on P.No.93)

Price under instalment plan

= Rs 210+ 3x260

= 210 +780 = Rs.990

Interest charged under instalment plan

= 990 – 970

= Rs 20

Principal of 1st instalment = Rs 970–210

P_1 = Rs 760

Principal of 2nd instalment = 760- 260

P_2 = Rs.500

Principal of 3rd instalment = 500-260

P_3 = Rs. 240

Total of Principal for 1 month = 760+500+240

= Rs 1500

Interest = Rs 20

we know that

$$S.I= \frac{P\times r\times t}{100}$$

Here S.I= Rs 20

P= Rs 1500

$$t = \frac{1}{12}$$

r=?

$$20= \frac{1500\times r\times 1}{100\times 12}$$

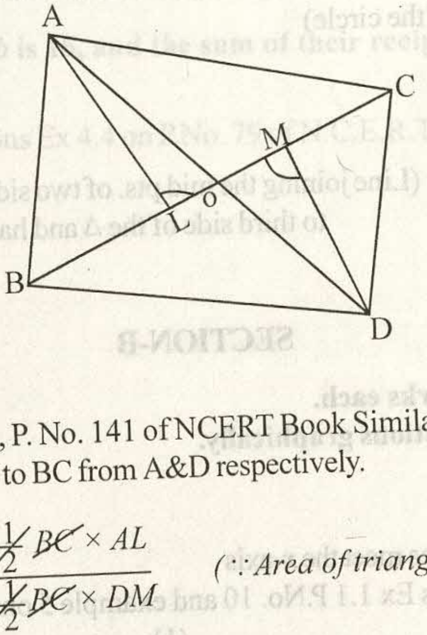
$$\frac{\overset{4}{\cancel{20}}\times 100\times \overset{4}{\cancel{12}}}{\underset{\cancel{5}\;1}{\cancel{1500}}\times 1} = r$$

4×4= 16% = *r*

Hence rate of interest is 16% Per Annum

9. In the figure, Δ ABD and Δ DBC are on the same base BC. AD and BC interesect at O. Prove that.

$$\frac{area\ (\Delta ABC)}{area\ (\Delta DBC)} = \frac{AO}{DO}$$



Answer:

See (Chapter 8, Ex. 8.3, Q.No. 5, P. No. 141 of NCERT Book Similar triangles)

Draw AL and DM perpendicular to BC from A&D respectively.

$$\frac{Area\ (\Delta ABC)}{Area\ (\Delta DBC)} = \frac{\cancel{\frac{1}{2}}\cancel{BC}\times AL}{\cancel{\frac{1}{2}}\cancel{BC}\times DM} \qquad (\because Area\ of\ triangle = \frac{1}{2}\,base\times height)$$

$$= \frac{AL}{DM} \qquad \text{———} (1)$$

In Δ AOL and Δ DOM

∠AOL = ∠DOM (vertically opp. L's)

∠ALO = ∠DMO (Each 90^0)

∠ALO ~ ∠DOM (A.A. corollory)

$$\therefore \frac{AL}{DM} = \frac{AO}{DO} \quad \text{\_\_\_\_\_\_\_\_} (2)$$

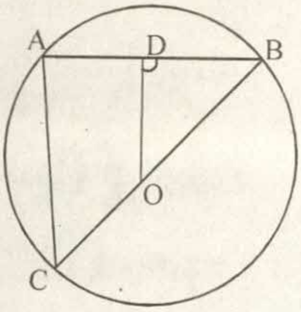
from (1) and (2) we get,

$$\frac{\text{Area of } (\Delta ABC)}{\text{Area } (\Delta DBC)} = \frac{AL}{DM} = \frac{AO}{DO}$$

Hence Proved

10. OD is perpendicular to a chord AB of a circle whose centre is O. If BC is a diameter, prove that CA= 2 OD

Answer 10: See chapter No.9, Circles Ex 9.1 on P.No. 10, Q.No. 8



Given : OD ⊥AB

To prove: CA= 2OD

Construction: Join AC

Proof ∵ OD ⊥ AB

∴ AD = DB (⊥to a chord from cnetre of the circle bisects the chord)

also BO= OC (radius of the circle)

In ΔABC

$$\frac{BD}{DA} = \frac{BO}{OC}$$

∴ OD 11 CA and OD = ½ CA (Line joining the mid pts. of two sides of B Δ is parallel to third side of the Δ and half of it.)

∴ CA = 2O D

Hence proved.

SECTION-B

Question number 11 to 20 carry 4 marks each.

11. Solve the following system of equations graphically.

$x + 2y = 5$

$2x = 3y = -4$

Also find the points where the lines meet the *x*-axis

Answer: See Chapeter No.1 linear eq^{n}s Ex 1.1 P.No. 10 and example 5 on P.No. 9 NCERT Book

$x + 2y = 5 \Rightarrow x = 5-2y$ ________ (1)

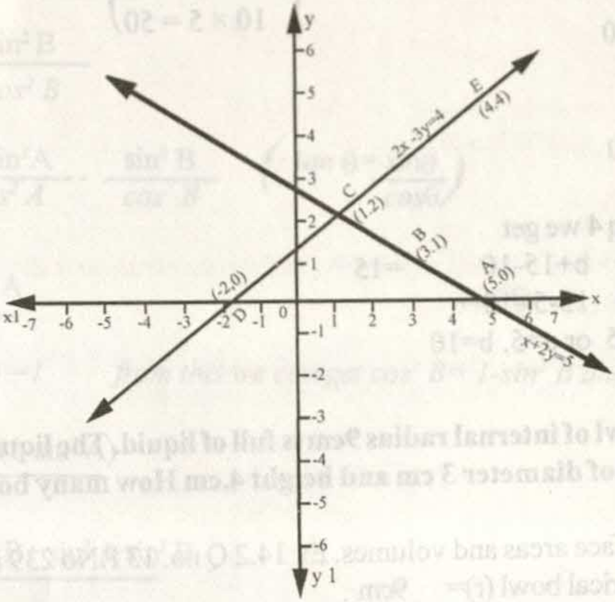
x	5	3	1
y	0	1	2

$2x- 3y = -4 \Rightarrow 2x = -4 + 3y$

$$\Rightarrow x = \frac{-4 +3y}{2} \quad \text{\_\_\_\_\_\_\_\_} (2)$$

x	-2	1	4
y	0	2	4

(**Note:** Draw the graph neatly with sharp pencil and scale. Mark the points accurately on graph. Take equal distance between the numbers on both the axis.)



Line ABC represents the graph of the eq^n x + 2y = 5 and the line DCE represents the graph of the eq^n 2x-3y-4

The two lines intersect at (1,2)

$\therefore$ The required solution is x = 1, y= 2.

The line ABC representing eq^n x+2y = 5 meets x- axis at (5,0) A and the line 2x-3y = y meets x-axis at D(-2,0)

12. The sum of two numbers *a* and *b* is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the numbers *a* and *b*.

Answer: See chapter 4 Quadratic equations Ex 4.4 on P.No. 79 of N.C.E.R.T Book.

Given a+b = 15 __________ (1)

$$\frac{1}{a} + \frac{1}{b} = \frac{3}{10}$$

Taking L.C.M. we get

$$\frac{a+b}{ab} \rightleftarrows \frac{3}{10}$$

By crossmultiplying we get

(a+b) 10 = 3ab

10a + 10 b = 3ab __________ (2)

from eq^n (1) we get

b= 15-a __________ (3)

from (3) put the Vaule of b in eq^n(2)

10a+ 10 (15-a) = 3a (15-a)

$\cancel{10}a + 150 - \cancel{10}a = 45a - 3a^2$

$-3a^2 + 45a - 150 = 0$ __________ (4)

Taking -3 common from eq^n (4 we get)

$-3(a^2-15a+50) = 0$

$a^2-15a+50 = 0$ (By splitting the middle term we get,)

$a^2-(10+5)a+50=0$

$a^2-10a-5a+50=0$ $\left(\begin{array}{l}\because 10+5=15\\ 10\times 5=50\end{array}\right)$

(a2- 10a) + (-5a +50) = 0

a(a-10)- 5(a-10)= 0

(a-10) (a-5)= 0

a-10=0 or a-5=0

a= 10 or a=5

By putting value of a in eq 4 we get

for a=10 b= 15-a= b+15-10 =15

for a=5 b= 15-a= 15-5=10

Hence a=10, b=5 or a=5, b=10

∴ number are 10 and 5

13. A hemispherical bowl of internal radius 9cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of diameter 3 cm and height 4.cm How many bottles are needed to empty the bowl?

Answer: See chapter 14 surface areas and volumes. Ex 14.2 Q no. 13 P.No 239 in N.C.E.R.T Book.

Radius of hemispherical bowl (r)= 9cm

Volume of hemispherical

bowl $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi (9)^3 Cu.Cm$ —①

Radius of cylindrical bottle (R) = $\frac{3}{2}$ *cm*

and height of bottle (h) = 4cm

∴ *Volume of Liquid in one bottle* = $\pi r^2 h$

$$=\pi\left(\frac{3}{\cancel{2}}\right)^2\times\cancel{4}$$

$$=\pi\frac{9}{\cancel{4}}\times\cancel{4}$$

$$=9\pi\ cu\ cm.$$

From (1, & (2)

number of bottles = $\dfrac{\textit{volume of hemisphere}}{\textit{volume of cylindical bottle}}$

$$=\frac{\frac{2}{3}\pi 9^3}{9\pi}$$

$$=\frac{2\times\cancel{9}\times\overset{3}{\cancel{9}}\times 9}{\cancel{3}\ \ \cancel{9}}$$

$$=2\times 3\times 9=54$$

Hence number of bottles needed is 54

14. Prove that

$$\tan^2 A-\tan^2 B=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}$$

or

Find the value of

$$\frac{\tan\theta.\cot(90^0-\theta)+\sec\theta.\text{cosec}(90^0-\theta)+\sin^2 35^0+\sin^2 55^0}{\tan 10^0 \tan 20^0 \tan 30^0 \tan 70^0 \tan 80^0}$$

Answer: see chapter No.12 Trignometry Ex 12.1 P.No 215/216/217 in N.C.E.R.T Book.

$$\tan^2 A-\tan^2 B=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}$$

$$L.H.S \quad \tan^2 A-\tan^2 B=\frac{\sin^2 A}{\cos^2 A}-\frac{\sin^2 B}{\cos^2 B} \quad \left(\because \tan\theta=\frac{\sin\theta}{\cos\theta}\right)$$

Taking L.C.M we get

$$\frac{\sin^2 A\cos^3 B-\sin^2 B\cos^2 A}{\cos^2 A\cos^2 B} \text{ ——① }$$

(we know $\sin^2\theta+\cos^2\theta=1$ from this we can get $\cos^2 B=1-\sin^2 B$ and $\cos^2 A=1-\sin^2 A$ in (1) we get)

$$\frac{\sin^2 A(1-\sin^2 B)-\sin^2 B(1-\sin^2 A)}{\cos^2 A\cos^2 B}$$

$$\frac{\sin^2 A-\sin^2 \cancel{A}\sin^2 B-\sin^2 B+\sin^2 \cancel{A}\sin^2 B}{\cos^2 A\cos^2 B}$$

$$\frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}=R.H.S.$$

Hence proved

or

See chapter 12 trignometry Ex 12.2 P.No 221

$$\frac{-\tan\theta.\,\text{coι}(90^0-\theta)+\sec\theta\,\text{coses}(90^0-\theta)+\sin^2 35^0+\sin^2 55^0}{\tan 10^0 \tan 20^0 \tan 30^0 \tan 70^0 \tan 80^0}$$

We know that $\cot(90^0-\theta)=\tan\theta$

$\text{Cosec}(90^0-\theta)=\sec\theta$

$\tan 30^0=\frac{1}{\sqrt{3}}$

Putting values of $\cot(90^0-\theta)$, *coses* $(90^0-\theta)$ *and* $\tan 30^0$ *in (1) we get.*

$$\frac{-\tan\theta.\tan\theta+\sec\theta\sec\theta+\sin^2 35^0+\sin^2 55^0}{\tan 10^0 \tan 20^0 \frac{1}{\sqrt{3}} \tan 70^0 \tan 80^0}$$

$$\frac{-\tan^2\theta.+\sec^2\theta+\sin^2 35^0+\sin^2(90-35^0)}{\tan(90-80)\tan(90-70)\frac{1}{\sqrt{3}} \tan 70^0 \tan 80^0}$$

We know that $\sin(90-\theta)=\cos\theta \therefore \sin(90-35^0)=\cos 35^0$

$\tan(90-\theta)=\cot\theta \therefore \tan(90-80)=\cot 80$ *and*

$\tan(90-70)=\cot 70^0$

$$\frac{(-\tan^2\theta.+\sec^2\theta)+(\sin^2 35^0+\cos^2 35)}{\cot+80^0\cot 70^0\frac{1}{\sqrt{3}} \tan 70^0 \tan 80^0} \text{ ------- (2)}$$

we know that $1+\tan^2\theta=\sec^2\theta$

$1=\sec^2\theta-\tan^2\theta=-\tan 2\theta+\sec 2\theta$

and $\sin^2\theta+\cos^2\theta=1 \therefore \sin^2 35^0+\cos^2 35^0=1$

$\cot 80^0 = \dfrac{1}{\tan 80^0}$ and $\cot 70^0 = \dfrac{1}{\tan 70^0}$

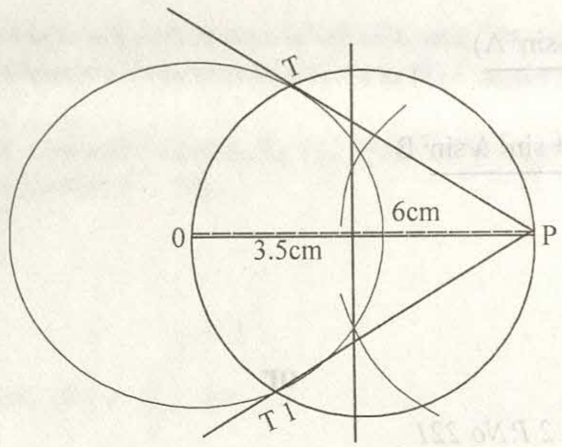
putting the values in (2) we get.

$$\frac{1+1}{\dfrac{1}{\cancel{\tan 80^\circ}} \times \dfrac{1}{\cancel{\tan 70^\circ}} \times \dfrac{1}{\sqrt{3}} \times \cancel{\tan 70^\circ}\,\cancel{\tan 80^\circ}} = \frac{2}{\sqrt{3}} = 2\sqrt{3}$$

Answer = $2\sqrt{3}$

15. Draw a circle of radius 3.5cm. From a point P outside the circle at a distance of 6 cm from the centre of circle, draw two tangents to the circle.

Answer: see chapter No.11 Geometrical construction, construction No.11.2 P.No 200 of N.C.E.R.T Book.



PT and PT^1 are required tangents

16. Find the value of *x* such that PQ=QR where the coordinates of P, Q and R are (6,-1); (1,3) and (*x*,8) respectively.

or

Find a point on x-axis which is equidistant from the points (7,6) and (-3,4).

Answer: See Chapter 16 co-ordinate Geometry Ex 16.2 P.No 284/85

Give Coordinates of P= (6,-1)
Coordinates of Q= (1,3)
Coordinates of R= (x,8)

We are Given PQ=QR

$\therefore PQ^2 = QR^2$ ——— (A)

using distance formula we get

$(x_2-x_1)^2 + (y_2-y_1)^2 = l^2$

$PQ^2 = (1-6)^2 + (3-(-1))^2$

$= (-5)^2 + (3+1)^2$

$= 25 + 4^2 = 25 + 16 = 41$ ——— (1)

$QR^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$= (x-1)^2 + (8-3)^2$

$= (x-1)^2 + 5^2$

$= (x-1)^2 + 25$ ——— (2)

Putting the values of PQ^2 *&* QR^2 *from (1)&(2) in (A) we get.*

$41=(x-1)^2+25$

$41-25=(x-1)^2$

$16=(x-1)^2$

$\pm\sqrt{16}=x-1$

$(x-1)=\pm 4$

$x-1=+4$ or $x-1=-4$

$x=4+1$ or $x=-4+1$

$x=5$ or $x=-3$

Or

Let P be a point on x-axis. As y coordinates of points on x-axis are always o.

$\therefore$ *let P be a point (x,o).*

let A (7,6) and B(-3,4) be the given points

$\therefore$ *P is equidestant from A & B*

$\therefore\ PA=PB$

or $PA^2=PB^2$ ———— (A)

Now by distance formula

$PA^2=(7-x)^2+(6-o)^2$

$=(7-x)^2+6^2$

$=(7-x)^2+36$ ———— (1)

$PB^2=(-3-x)^2+(4-o)^2$

$=(-3-x)^2+4^2$

$=(-1)^2(3+x)^2+16$

$=(3+x)^2+16$ ———— (2)

From (1) & (2) putting values of PA^2 *&* PB^2 *in A we get*

$(7-x)^2+36=(3+x)^2+16$

$(7-x)^2-(3+x)^2=16-36$

$(49+x^2-14x)-(9+x^2+6x)=-20$

$49+\cancel{x^2}-14x-9-\cancel{x^2}-6x=-20$

$40-20x=-20$

$-20x=-20-40$

$+20x=+60$

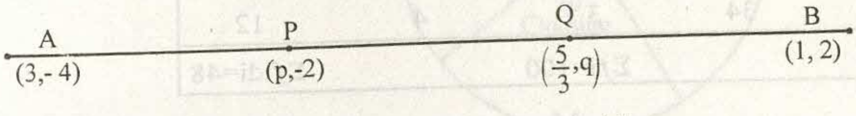
$x=\frac{\overset{3}{\cancel{60}}}{\cancel{20}}$

$x=3$

$\therefore$ *coordinates of p=(3,0)*

17. The line- segment joining the points (3,-4) and (1,2) is trisected at the points P and Q. If the coordinates of P and Q are (p,-2) and $\left(\frac{5}{3},q\right)$ respectively, find the values of *p* and *q*.

Answer: see chapter 16 coordinate Geometry Ex 16.3 on P.No 290-91 of N.C.E.R.T Book.



As P and q trisect line segment AB

$\therefore$ AP : PB= 1:2

By section formula $\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_1 + m_2 y_1}{m_1 + m_2}$

$$\text{Point } P = \left(\frac{1 \times 1 + 2x3}{1+2}, \quad \frac{1 \times 2 + 2 \times (-4)}{1+2} \right)$$

$$= \left(\frac{1+6}{3}, \quad \frac{2-8}{3} \right) = \left(\frac{7}{3} \frac{-6}{3} \right) = \left(\frac{7}{3}, -2 \right)$$

But Given P= (p,-2)

$$(p,-2) = \left(\frac{7}{3}, -2 \right)$$

$$\therefore p = \left(\frac{7}{3} \right)$$

Again AQ : QB = 2:1

$$\text{Point } Q = \left(\frac{2 \times 1 + 1 \times 3}{2+1}, \quad \frac{2 \times 2 + 1 \times (-4)}{2+1} \right)$$

$$\left(\frac{2+3}{3}, \frac{4-4}{3} \right) = \left(\frac{5}{3}, 0 \right)$$

But Given $Q = \left(\frac{5}{3}, q \right)$

$$\therefore \left(\frac{5}{3}, q \right) = \left(\frac{5}{3}, 0 \right)$$

$$\therefore q = 0$$

Hence $p = \frac{7}{3}, \; q = 0$

18. Find the mean of the following distribution:

Class	Number of Students
4-8	2
8-12	12
12-16	15
16-20	25
20-24	18
24-28	12
28-32	13
32-36	3

Answer: See chapter No.15 Statistics Ex 15.2 P.No. 267-68 N.C.E.R.T. Book.

Class	xi	fi	$di = \frac{xi-18}{4}$	fidi
4-8	6	2	-3	-6
8-12	10	12	-2	-24
12-16	14	15	-1	-15
16-20	18	25	0	0
20-24	22	18	1	18
24-28	26	12	2	24
28-32	30	13	3	39
32-36	34	3	4	12
Total		Σfi=100		Σfidi=48

Let assumed mean a=18

Here $\Sigma fi = 100$ & $\Sigma fidi = 48$ Class interval=4 (c)

$$\text{mean} = \overline{x} = A + \frac{\Sigma fidi}{\Sigma fi} \times c$$

$$= 18 + \frac{48}{\underset{25}{\cancel{100}}} \times \overset{1}{4}$$

$$= 18 + 1.92 = 19.92$$

$\therefore$ required mean $\overline{x} = 19.92$

19. Given below is the expenditure of a person on different items out of his salary of Rs. 14,400.

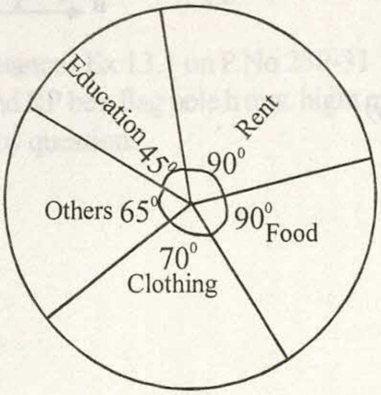
Item	**Clothing**	**Food**	**Rent**	**Education**	**Others**	**G. Total**
Expenditure (in Rupees)	2,800	3,600	3,600	1,800	2,600	14,400

Draw a pie- chart to depict the above data.

Answer: See chapter 15 statistics Ex No.15 Q No.6 on P.No 255-56 in N.C.E.R.T. Book

Item	Expenditure (Rs)	central	Angle
Clothing	2800	$\frac{\overset{147}{\cancel{2800}}}{\underset{\cancel{72}\,\cancel{36}}{\cancel{14400}}} \times \overset{10}{\cancel{360}}$	70^0
Food	3600	$\frac{\overset{4}{\cancel{3600}}}{\underset{\cancel{16}\,\cancel{4}}{\cancel{14400}}} \times \overset{9}{\cancel{360}}$	90^0
Rent	3600	$\frac{\overset{4}{\cancel{3600}}}{\underset{\cancel{16}\,\cancel{4}}{\cancel{14400}}} \times \overset{9}{\cancel{360}}$	90^0
Education	1800	$\frac{\overset{2}{\cancel{1800}}}{\underset{\cancel{16}\,\cancel{8}\,\cancel{4}\,1}{\cancel{14400}}} \times \overset{45}{\overset{\cancel{180}}{\cancel{360}}}$	45^0
Other	2600	$\frac{\overset{13}{\cancel{2600}}}{\underset{\cancel{72}\,\cancel{8}}{\cancel{14400}}} \times \overset{5}{\overset{\cancel{40}}{\cancel{360}}}$	65^0
Total	14400		360^0

The required pie chart is given below.



20. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the card drawn is neither a red card nor a queen.

Answer: See chapter No.15 Statistics Ex 15.3 on P.No 273-274 in N.C.E.R.T Book.

Give Total Cards = 52

Total red cards in a deck=26

∴ No. of favourable outcomes=26

Total No. of possible outcomes= 52

$$\therefore \text{P (a red card)} = \frac{\text{No. of favorable out comes}}{\text{Total No. of possible out comes}}$$

$$\text{p (queen)}= \frac{\text{No. of favorable out comes}}{\text{Total No. of possible out comes}}$$

$$= \frac{2}{52} \qquad \text{——— (2)}$$

Form (1) & (2) we get.

$$\text{P (Either a red card or a queen)} = \frac{26}{52} + \frac{2}{52} = \frac{28}{52}$$

$$\text{so P (neither a red card nor a queen)} = 1 - \frac{28}{52}$$

$$= \frac{52-28}{52} = \frac{\cancel{24}^{\cancel{12}^{6}}}{\cancel{52}_{\cancel{26}_{13}}}$$

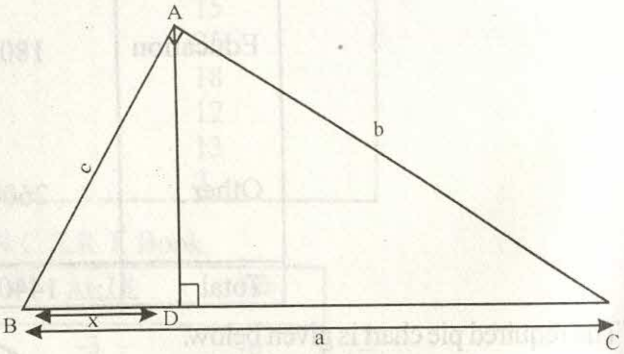
$$= \frac{6}{13}$$

$$\text{Hence required probability} = \frac{6}{13}$$

SECTION -C

21. Prove that in a right angled triangle the square on the hypotenuse is equal to sum of the squares on other two sides.
Using the above result, prove that the sum of squares on the sides of a rhombus is equal to sum of squares on its diagonals.

Answer: See chapter No. 8 similar triongles Ex No. 8.4. P.No 145 and thin 8.3 on P.No 141-42 in N.C.E.R.T Book



Given: A right DABC in which $\angle A = 90^0$

To Prove: $BC^2 = AB^2 + AC^2$

Construction: From A draw $AD \perp BC$

Proof: Let $AB=c$, $AC= b$, $BC= a$ *and* $BD = x$

then $DC= a-x$

In $\Delta\, ABC \;\&\; \Delta\, DBA$

$\angle BAC = \angle BDA$ *(Each* 90^0*)*

$\angle ACB = \angle DAB$ *(* $90- \angle B$ *)*

$\Delta\, ABC \sim \Delta\, DBA$ (A.A *corollary)*

$$\frac{BC}{BA} = \frac{AB}{DB}$$

$$\frac{a}{c} = \frac{c}{x}$$

$$a\,x = c^2 \text{ ———— } (1)$$

Similary $\Delta ABC \sim \Delta DAC$

$$\frac{AC}{DC} = \frac{BC}{AC}$$

$$\frac{b}{a-x} = \frac{a}{b}$$

or $b^2 = a\,(a-x) = a^2 - ax$ ———— (2)

Adding (1) & (2) we get

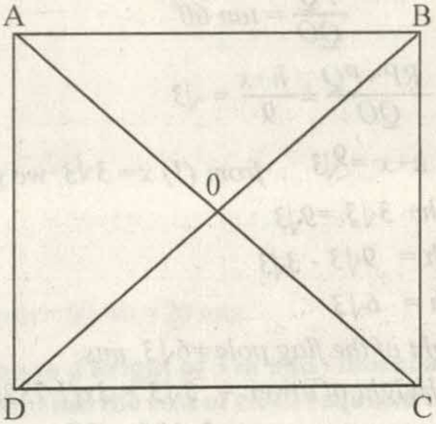
$$b^2 + c^2 = ax + a^2 - ax$$

$$b^2 + c^2 = a^2$$

Hence $AB^2 + AC^2 = BC^2$

Hence Proved

Rider:



Let ABCD be a rhombus. Its diagonals AC& BD meet at O. We have to prove that $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

The diagonals of a rhombus bisect each other at right angles

$\therefore AC \perp BD$

In $\Delta OAB,\ AB^2 + A0^2 + 0B^2$

or $AB^2 \left(\frac{AC}{2}\right)^2 + \frac{BD}{2}\Big)^2$

$$AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$$

or $4\,AB^2 = AC^2 + BD^2$ —— (1)

But $4\,AB^2 = AB^2 + AD^2 + BC^2 + DC^2$ —— (2)

($\because$*ABCD is a Shombus gives* $AB = BC = CD = DA$)

From (1) and (2) we get

$$AB^2 + AD^2 + DC^2 + BC^2 = AC^2 + BD^2$$

Hence Proved

22. **On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60^0 and 300 respectively. Find the height of the tower and flag pole mounted on it.**

Or

Form a building 60 metres high the angle of depression of the top and bottom of lamppost are 300 and 600 respectively. Find the distance between lamppost and building. Also find the difference of height between building and lamppost.

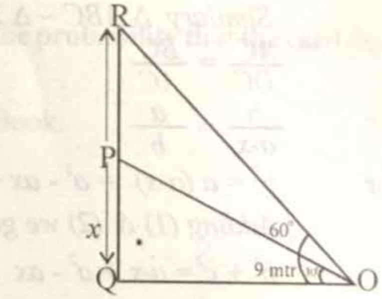
Answer: See chpater No 13. Height & distances Ex 13.1 on P.No 230-31

Let PQ be a tower of height x mts and RP be a flag pole h mts. hight mounted on the tower. Let o be a point 19mts. away from foot of the tower. Then by question.

$\angle POQ = 30^0$

$\angle ROQ = 60^0$

In right angled Δ PQO



$$\frac{PQ}{QO} = \frac{Perp}{Base} = tan\ 30^{\circ} \qquad \frac{x}{9} = \tan 30^{\circ}$$

$$x = \frac{9\times 1}{\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{\overset{3}{\cancel{9}}\sqrt{3}}{\cancel{3}} = 3\sqrt{3}\,m \text{ ——— (1)}$$

$\therefore$ *Height of the tower = PQ =x* $3\sqrt{3}$ *mts.*

In right angled ΔRQO

$$\frac{PQ}{QO} = tan\ 60^{\circ}$$

$$\frac{PQ}{QO} = \frac{RP+PQ}{QO} = \frac{h+x}{9} = \sqrt{3}$$

$h+x = 9\sqrt{3}$ *from (1)* $x= 3\sqrt{3}$*. we get*

$h+ 3\sqrt{3} = 9\sqrt{3}$

$h = 9\sqrt{3} - 3\sqrt{3}$

$h = 6\sqrt{3}$

$\therefore$ *Height of the flag pole=*$6\sqrt{3}$ *mts.*

Hence Height of tower = $3\sqrt{3} = 3x(1.732)$

$= 5.196 = 5.2$ *mts.*

Height of the flag pole=$6\sqrt{3} = 6x(1.732)$

$= 10.392 = 10.4$*mts.*

or

Let PQ be the building 60 m high. let AB be the lamp post of height h mts.
Let distance between lamp post & building be *x* mts.

Thus BQ = AR= x mts.

P R= PQ – RQ= (60-h) mts.

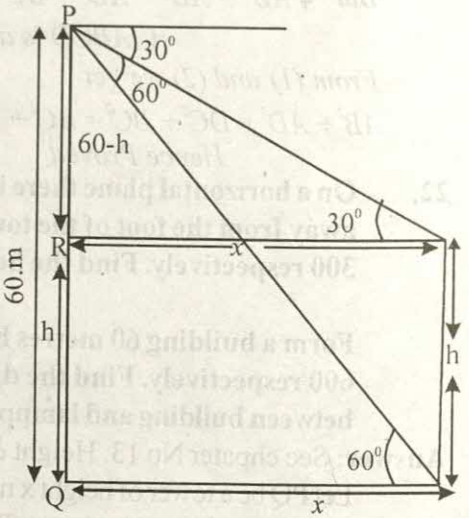
$\angle APR= 30^{\circ}$ & $\angle PBQ=60^{\circ}$

In right angled Δ PQB

$$\frac{PQ}{QB} = \tan 60^{\circ}$$

$$\frac{60}{x} = \sqrt{3}$$

$$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{60}\sqrt{3}}{\cancel{3}} = 20\sqrt{3}$$



Distance between lamp post & building

= 20x 1.732

= 34. 640

= 34. 64 mts.

In right angled $\Delta PRA = \frac{PR}{AR} = \tan 30^\circ$

$$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

$\sqrt{3}\ (60-h) = x$ (By Cross multiplying)

$60\sqrt{3} - \sqrt{3}h = x$

$\sqrt{3}\ (60-h) = x$

$$60-h = \frac{x}{\sqrt{3}}$$

$$60-h = \frac{x \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{x\sqrt{3}}{3}$$

$$60-h = \frac{20\sqrt{3} \times \sqrt{3}}{3} = \frac{20 \times \cancel{3}}{\cancel{3}} = 20$$

$-h = 20 - 60$

$+h = +40$

$h = 40$ mts.

Height of lamp post = 40 mts

Difference between height of building & that of lamp post = 60-40 = 20 mts.

23. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of Rs. 80 per sq. m.

or

The radii of circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10cm. Find its total surface area.

Answer:

See Chapter No .14 Surface Areas and Volumes Ex 14.3 on P.No. 247 in N.C.E.R.T Book

Radius of base (r) = 14 mts.

Height of cylindrical portion (h) = 3mts

Total height of tent = 13.5 mts

$\therefore$ Height of conical portion = 13.5 - 3.0 $\left(\begin{array}{r}13.5\\ -\ 3.0\\ \hline 10.5\end{array}\right)$

= 10.5 m

Let l be the slant height of the conical section

$$\therefore \quad l = \sqrt{(14)^2 + (10.5)^2}$$

$$= \sqrt{196 + 110.25}$$

$$= \sqrt{306.25}$$

$$17.5m$$

$\therefore$ Cloth required for the tent = cloth for curved surface of the cylindrical portion + cloth for curved surface of conical portion

$= 2\pi r h + \pi r l$

$= \pi r\ (2h + l)$

$=\frac{22}{7}\times\overset{2}{\cancel{14}}(2\times3+17.5)$

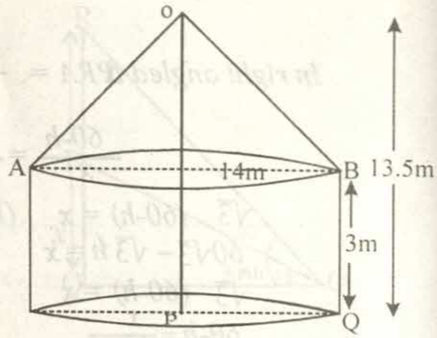
$=22\times2\,(6+17.5)$

$=44\,(23.5)$

$=1034.\ m^2$

Rate of cloth = 1034×80

= 82, 720 Rs.



Or

See chapter No.14 Surface Areas & Volumes Ex 14.3 on P.No 247.

Given R = 33 cm

r = 27 cm.

& slant height = 10 cm.

∴ Total surface area of solid frustum of cone.

$=\pi Rl+\pi rl+\pi R^2+\pi r^2$

$=\pi l(R+r)+\pi(R^2+r^2)$

$=\frac{22}{7}\times10\,(33+27)+\frac{22}{7}\left((33)^2+(27)^2\right)$

$=\frac{22}{7}\times\left(10\times60+(33)^2+(27)\right)^2$

$=\frac{22}{7}\times\left(600+1089+729\right)$

$=\frac{22}{7}\times2418$

$=\frac{53196}{7}$

$=7599.4$ sq cm.

24. If a line touches a circle and from the point of contact a chord is drawn, prove that the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments. Using the above theorem, prove the following:

P is mid point of arc APB. Prove that tangent QR drawn at P to the circle is parallel to AB.

Answer: See chapter No.10 Tangents to a circle. theorm No. 10.3 on P.No 189.

Give: PQ is a tangent to a circle with centre O at a print A, AB is a chord and C.D are points in the two segments of the circle formed by the chord AB.

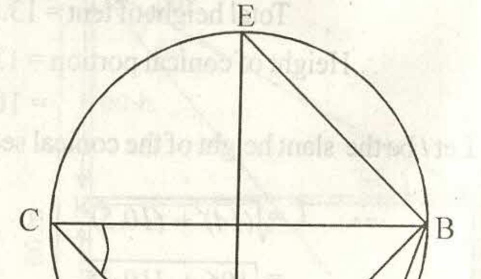
To Prove: ∠BAQ = ∠ACB

and ∠BAP = ∠ADB

Construction: Draw the diameter AOE and join EB

Proof: In triangle AEB

∠ABE = 90° (Angle in a semicircle)



∴ ∠AEB + ∠EAB = 90° ——— (1)

Again as EA⊥PQ,

∴ ∠EAB + ∠BAQ = ∠EAQ = 90° ——— (2)

From (1) and (2) we have

∠AEB = ∠BAQ

∠ACB = ∠AEB (Angle subtended in the same segment)

∴ ∠BAQ = ∠ACB

Next ∠BAQ + ∠BAP = 180° (Linear pair of L's)

and ∠ACB + ∠ADB = 180° (opposite L's of a cyclicquad)

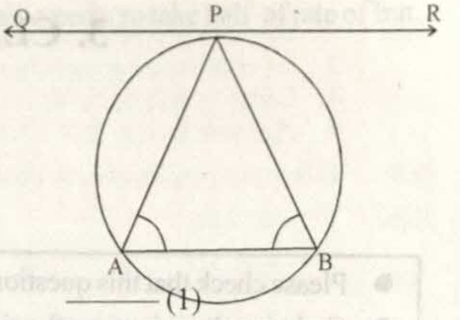
∴ ∠BAQ + ∠BAP = ∠ACB + ∠ADB

∠BAQ + ∠BAP = ∠BAQ + ∠ADB (∠BAQ = ∠ACB)

∴ ∠BAQ = ∠ADB

Hence Proved

Rider:



Given : P is the mid- point of arc APB. QPR is tangent to the circle at P.

To Prove: QPR 11 AB

Proof : $\because$ P is the mid point of $\widehat{APB}$

$\therefore$ $\widehat{AP} = \widehat{PB}$

$\therefore$ AP = PB

$\therefore$ $\angle PAB = \angle PBA$ (L's opp to the equal sides of a triangle are equal) ——— (1)

also $\angle QPA = \angle PBA$ (By above them) ——— (2)

From (1) & (2) we get.

$\angle QPA = \angle PAB$

But these are alternate QPR IIAB interior angles.

25. **The monthly income of Ruchi (age 30 years) is Rs. 18,200 (excluding HRA). She donates Rs. 8,400 towards Prime Minister Relief Fund (Relief 100%) and Rs. 12,000 to an orphanage (Relief 50%). She contributes Rs. 4,500 per month towards Provident Fund and Rs. 2,500 per quarter as LIC premium.She purchases. National Saving Certificates worth Rs, 8,000. She pays Rs. 900 per month as income-tax for the first 11 months. Calculate her income-tax liability for last month of the year.**

Answer: See chapter No. 7 Income tax Ex. 7.1 on P.No 113-14 in N.C.E.R.T Book.

Annual salary = Rs 18200 × 12
= 218400 Rs.
Donation to P.M relief fund = Rs 8400

Exemption for donation to orphanage = 12000 × $\frac{1}{2}$ = 6000 Rs.

$\therefore$ Total exemption on donation = Rs 14400 (8400 + 6000)
Balance of income = Rs 218400 - 14400
= Rs 204000

Standard Deduction = 30000
Taxable income = 204000-30000
= Rs. 174000

Income tax on Rs 174000 = Rs 19000 + 24000 $\frac{30}{100}$
= 19000+ 7200
= Rs. 26200

Savings: P.F. 4500× 12 = 54000
L.I.C. Premium = 2500×4 = Rs. 10,000
and NSC = Rs 8000
$\therefore$ Total Savings = Rs 72000 (54000 + 10000 + 8000)
Rebate in tax = 15% of Rs 72000 subject to a maximum of Rs 10,500
= Rs 10,800 subject to a maximum of Rs 10,500
= Rs 10, 500

Special rebate for women = Rs 5000
$\therefore$ Total rebate = 10,500+5000
= Rs 15500

$\therefore$ Payable income tax = 26200 - 15500
= Rs 10700

Education cess = 10700 × $\frac{2}{100}$
= Rs 214

$\therefore$ Total tax payable = 10700+214
= Rs 10914

Tax deducted in 11 months = 900×11
= 9900 Rs

$\therefore$ Income tax liability in the last month
= 10914 - 9900
= Rs. 1014

Library
B. C.

3. CLASS -X PAPER-II : SOLUTIONS

Candidates must write the code on the title page of the answer book.

- Please check that this question paper contains 12 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 25 questions.

MATHEMATICS

[Time Allowed: 3 Hours] *Maximum marks: 100*

General Instructions:

(i) All questions are compulsory.

*(ii) The question paper consists of 25 questions divided into three sections —**A, B and C**. Section **A** contains **10** questions of **3** marks each, Section **B** is of **10** questions of **4** marks each and section **C** is of **5** questions of **6** marks each.*

(iii) There is no overall choice. However, an internal choice has been provided in two questions of three marks each, two question of four marks each and two questions of six marks each.

(iv) In question on construction, the drawing should be neat and exactly as per the given measurements.

(v) Use of calculator is not permitted.

SECTION-A

Q.1 Express the following as a rational expression in lowest terms:

$$\frac{x^3-125}{x^2-4x-5}\times\frac{x^2-1}{x^2+x-2}\div\frac{x^2+5x+25}{x^2+4x+4}$$

1. Apply a^3-b^3 formulae for factorization of x^3-125 [ie-$a^3-b^3=(a-b)(a^2+ab+b^2)$].
2. Apply a^2-b^2 formulae or factorization of x^2-1 [($a^2-b^2=(a-b)(a+b)$].
3. for factorization of x^2-4x-5; x^2+x-2 & x^2+4x+4, use the splitting of middle term.
4. Replace the division singly multiplication sign by reciprocating the last rational expression.
5. Answer $x+2$

Q.2 Find the 6th term from end of the A.P. 17,14,11, ..., -40

1. First reverse the given A.P. such that a= -40 and d= +3 and last term will be 17.
2. By this it will be easy to get the 6th term from the end.
3. Apply the formulae and get answer.
4. Answer 6th term form end = -25

Q.3 A loan of Rs. 10,815 is to be returned in three equal hald-yearly instalments. Calculate the amount of each instalment, if the rate of interest is $13\frac{1}{3}$% Per annum, compounded half-early.

1. B Let each installment be Rs.x.
2. Apply compound interest formulae. $A=P\left(\frac{1+r}{100}\right)^n$
   Where A= amount, P=Principal, r=rate of intt.

3. The rate of intt is given for full year but the instalment has to be paid half- yearly so take half of rate of Intt.

i.e $r=\frac{40}{3}\%\ \frac{1}{2}$ of $r=\frac{40}{3}\times\frac{1}{2}=\frac{20}{3}\%$

4. Same as Q.No.-7 in Set1.
5. Answer Rs. 4096.

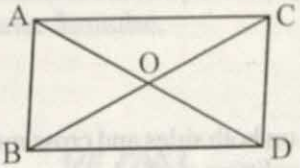
Q.4 Find the L.C.M. of the following polynomials:

$2x^3-432$; $2x^2-11x-6$; x^2-36

1. In $2x^3-432$. Take 2 common $2(x^3-216)$
   Now find factors of x^3-216 as x^2-63
   $= a^3-b^3= (a-b)\ (a^2+ab+b^2)$
2. In x^2-36 use formulae $a^2-b^2=(a-b)(a+b)$
3. In $2x^2-11x-6$ use splitting of middle term method.
4. L.C.M. of $(2x^3-432)$; (x^2-36); $(2x^2-11x+6)$
   $2(x-6)\ (x+6)(2x+1)\ (x^2+6x+36)$

Q.5. In the Figure, ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that

$$\frac{\text{area }(\Delta\ ABC)}{\text{area }(\Delta\ DBC)}=\frac{AO}{DO}$$



Ans. 1. Construct altitudes from A & D on diagonal BC.
2. Find the ratio of the areas of given triangles.
3. Same as Q.No.9 of Set 1.

Q.6 Find the number of terms of A.P. 64, 60, 56, ... so that their sum is 544.

Ans. 1. Find, a, d and then apply the sum formulae for A.P.

2. $S_n=\left[\frac{n}{2}2a+(n-1)d\right]$

   Find n by substituting the values of S_n, a and d.

3. Ans. n= 16 or 17

Q.6

or Part

If the *n*th term of an AP is $(2n+1)$, find the sum to frist *n* terms of the A.P.

Ans. 1. Take n= 1, 2, 3 and so on and find A.P.

2. Find a and D from the A.P.
3. Apply sum formulae

   $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

   And find the sum of n terms of A.P.

4. Same as Q.No. 6 in Set 1.
5. Same of 1st terms $= n^2+2n$

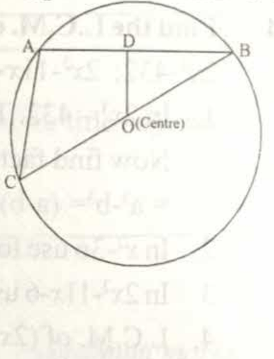
Q.7 Solve the following system of linear equations:

$ax+by=a-b$

$bx-ay=a+b$

Ans. 1. By multiplying eq^n (1) by 'a' and eq^n (2) by 'b'.
2. By applying elimination method by equating coefficient solve for x and of.
3. Same as question No. 3 of set 1.
4. Answer $x=1, y=-1$

Q.8 OD is perpendicular to chord AB of a circle whose cnetre is O. If BC is a diameter, prove that CA= 2 OD



Ans. 1. Join A&C.

2. $OD \perp AB \therefore AD = DB$

Perpendicular from centre of the circle to chord, always bisects the chord.

3. Now use the result that live joining the mid pts, of two sides of a triangle is parallel to the third side and $\frac{1}{2}$ of it.

$OD= \frac{1}{2}$ CA

CA= 2 OD Hence Proved

4. Same as Q.No.-10 of Set 1.

Q.9 Solve for *x*:

$$\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x};\ a \neq 0,\ b \neq 0,\ x \neq 0$$

or

Solve of *x* :

$abx^2 +(b^2-ac)\,x- bc = 0$

Ans. 1. Take all terms of *x* one side
2. Now solve for *x* by taking L.C.M. on both sides and cross multiplying.
3. Solve the equation by middle term splitting method.
4. Same as Q.No.-5 of set 1,
5. Answer $x = -a$ or $-b$

Or Part

1. Open the breaket $(b^2-ac)x$
2. Apply middle term splitting method. And find the value of *x*.
3. Same as Q.No. 5 of Set1.
4. $x = \frac{-b}{a}, \frac{c}{b}$

Q.10. A fan is available for Rs. 970 cash or Rs. 210 as cash down payment followed by three equal montly instalments of Rs. 260 each. Find the rate of interest charged under the instalement plan.

Ans. 1. Find the price under instalement plan to get the interest.
2. Find the principal for the three instalement.
3. Find Rate of intt. by applying simple interest formulae. $S.I = \frac{P \times r \times t}{100}$
4. Same as Q.No. 8 of Set 1,
5. $r = 16\%$

SECTION –B

Q.11 Solve the following system of equation graphically:

$x-2y =-3$

$2x+y = 4$

Ans. 1. Find three values of *x* and *y* for both equation.
2. Plot the graph with the help of sharp pencil and scale neatly.
3. Take equal distance between two numbers on both axis.
4. Write the points of both eq^{n}s. which lie on *x*- axis.

5. Write the solution of given eq^{n}s.

Q.12 The sum of two number is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Ans. 1. Form two equations for a & b with given conditions. (i) $a + b$ =16 & (ii) $\frac{1}{a} + \frac{1}{b} = \frac{1}{3}$

2. Solve the eq n s. by splitting the middle term method.
3. Write the numbers.
4. Answer Nos. 4 and 12. $a = 4$, $b = 12$

Q.13 Draw a circle of radius 3.5 cm. From a point P outside the circle at a distance of 6 cm from the centre of circle, draw two tangents to the circle.

Ans. 1. Draw a circle with r =3.5 cm.

2. Take a point P outside the circle such that O P= 6 cm.
3. Draw perpendicular bisector of O P.
4. Take the point where the perpendicular bisector meets O P as center draw another circle which toneless the circle with radius 3.5cm. at two points.
5. Now draw tangents from P to these two points.
   From this we get the required two tangents to a circle from the point outside the circle.
6. Same as Q.No.15 of Set 1.

Q.14 Find the value of x scuh that PQ = QR where the coordinates of P, Q and R are (9,-1), (1, 3) and (x, 8) respectively.

OR

Find a point on x-axis which is equidistant from the points (7, 6) and (-3, 4).

Ans. 1. Find the value of PQ and QR by distance formulae.

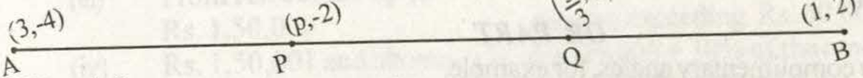
2. As PQ= QR is given
   Equate the values of PQ & QR
   And find the value of x.

OR PART

1. Point on x-axis will have coordinates (x,o).
2. Let A have coordinates (7,6)
   B " " (-3,4)
   C " " (x,o)
3. Now find AC & BC by distance formulae.
4. Given AC = BC. Equating values of AC & BC find the value of x.
5. Same as Q.No. (16 or part) of Set 1.
6. Answer x= 3.

Q.15 The line-segment jointing the points (3, -4) and (1, 2) is trisected at the points P and Q. If the coordinated of P and ! are (P, -2) and $\left(\frac{5}{3}, q\right)$ respectively, find the vales of p and q.

Ans.



AP; PB = 1:2

By using section formulae find the value of p.

$$\frac{m_1 \times x_2 + m_2 \times x_1}{m_1 + m_2} \quad where\ m_1 = 1,\ m_2 = 2.$$

$$\frac{m_1 \times y_2 + m_2 \times y_1}{m_1 + m_2}$$

Similarly AQ; QB = 2:1

And then by using section formulae find the value of a.

Same as Q.No. 17 of Set1. Aswer. $p = \frac{7}{3}, q = 0$

Q.16 Given below is the expenditure of a person on different items out of his salary of Rs. 14,400.

Item	**Clothing**	**Food**	**Rent**	**Eduction**	**Others**	**G.toal**
Expenditure(In Rs.)	2800	3,600	3,600	1,800	2,600	14,400

Draw a pie-chart to depict the above data.

Ans. Find the central angles of given data.

eg. For clothing $\frac{\overset{7}{\cancel{2800}}}{\underset{\cancel{164}}{\cancel{4400}}} \times \overset{\cancel{4}}{\cancel{300}} = 7\times10 = 70^0$

Like this find the angle for food, Rent, education and others and draw a neat pie chart with the help of sharp pencil, scale, compose and protector.

Same as Q.No.19 of Set 1.

Q.17. Find the mean of following distribution:

Class	Number of Students
4-8	2
8-12	12
12-16	15
16-20	25
20-24	18
24-28	12
28-32	13
32-36	3

Ans. Find Σ and x_1 of the given data and then mean of given data by using any one of the formulae for finding mean of grouped data. Same as Q.No. 18 of set 1. Ans. $x = 19.92$

Q.18 A hemispherical bowl of intrenal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and heigth 4cm. How many bottles are needed to empty the bowl?

Ans.
1. Find the volume of hemispherical bowl using $\frac{2}{3}\pi r^3$= vol of hemispherical bowl.
2. Find the volume of one cylindrical bottle = $\pi r^2 h$.
3. Now dividing the volume of hemispherical bowl by the volume of one cylindrical bottle.
4. Find the number of bottles needed to empty the bowl.
5. Same as Q.No. 13 of Set 1.
6. Answer No. of bootles = 54

Q19. Prove that

$$\tan^2\text{-}A\text{-}\tan^2 B = \frac{\sin^2 A\text{-}\ \sin^2 B}{\cos^2 A\ \text{co}^2 B}$$

OR

Find the vaule of

$$\frac{-\ \tan\theta.\cot(90^\circ\text{-}\theta) + \sec\theta.\ \text{cosec}\,(90^\circ\text{-}\theta) + \sin^2 35^\circ + \sin^2 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 30^\circ \tan 70^\circ \tan 80^\circ}$$

Ans. Using $\tan\theta \frac{\sin\theta}{\cos\theta}$ on L.H.S. of the identity prove that L.H.S = R.H.S.

Hence proved.

Same as Q.No.14 of set 1.

OR PART

Change all the necessary complimentary angles, for example.

$\cot(90^\circ\text{-}\ \theta) = \tan\theta^\circ$

$\text{coses}\,(90^\circ\text{-}\theta) = \text{Sec}\theta^\circ$

$\sin 55^\circ = \text{Sin}\,(90\text{-}35^\circ) = \cos 35^\circ$

$\tan 10^\circ = \tan(90^\circ\text{-}80^\circ) = \cot 80^\circ$

$\tan 20^\circ = \tan(90^\circ\text{-}70^\circ) = \cot 70^\circ$

Now final the value, Same as Q. No.-14 (or part) of set 1.

Q.20 Find the probability that a number selected from the numbers 1 to 25 is not a prime number when each of given number is equally likely to be selected.

Ans.
1. First find and write all the prime numbers between 1 to 25.
2. Subtract the P(of prime numbers) $\frac{8}{25}$ = from 1 (maximum probability for an outcome.)

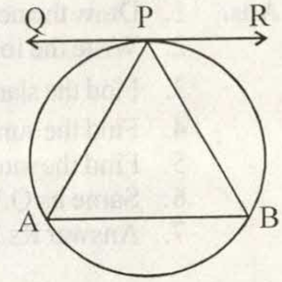
3. P(not a prime no.) = $1-\frac{8}{25}=\frac{17}{25}$

SECTION-C

Q.21. Prove that in a triangle, a line drawn parallel to one side to intersect the other two sides in distinct points, divides the two sides in the same ratio.
Using above prove that the quadrilateral ABCDE is a trapezium if the diagonal AC and BD of the quadrilateral ABCD intersect each other at O such that $\frac{AO}{OC}=\frac{BO}{OD}$

Ans.
1. Sec this 8.1 in chapter 8 similar Triangles. i.e. Basic Proportionally Thin or Thales thin on P.No. 118-119
2. Draw neat and correct figure.
3. Write proper reason while giving proof of the thin.
4. Use the result of the thim. In solving the rider second part of the question.

Q.22. If a line touches a circle and from the point of contact a chord is drawn, prove tht the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.
Using the above theorem, prove the following :
P is mid point of arc APB. Prove that tangent QR drawn at P to the circle is is parallel to AB.



Ans.
1. Sec thin 10.3 in chapter No. 10 Tangents to a Circle on P.No. 189-190
2. Draw neat and correct figure.
3. Write proper reasons while giving proof of the theorem.
4. Use the result of the thin in solving the rider in second part of the question.

Q. 23 The monthly income of Ruchi (age 30 years) is Rs. 18,200 (exculding HRA). She donates Rs. 8,400 towards Prime Minister Relief Fund (Relief 100%) and Rs. 12,000 to an orphanage (Relief 50%). She contributes Rs. 4,500 per month towards Provident Fund and Rs. 2,500 per quarter as LIC premium. She purchases National Saving Certificates worth Rs. 8,000. She pays Rs. 900 per month as income-tax for the first 11 months. Calculate her income-tax liability for last month of the year.

For calculating income tax, use the following:

(a) Standard Deduction (i) Rs. 30,000 if income is up to Rs, 5,00,000
(ii) Rs. 20,000 if income is more than Rs. 5,00,000

(b) Rates of Taxes

	Slab.	Income Tax
(i)	Up to Rs. 50,000	No tax
(ii)	From Rs. 50,001 up to Rs. 60,000	10% of the amount exceeding Rs. 50,000
(iii)	From Rs. 60,001 up to Rs. 1,50,000	Rs. 1,000 + 20% of the amount exceeding. Rs. 60,000
(iv)	Rs. 1,50,001 and above	Rs. 19,000 + 30% of the amount exceeding Rs. 1,50,000

(c) Rebate in Tax
(i) 20% of the saving subject to a maximum of Rs. 14,000 if the gross income is up to Rs. 1,50,000
(ii) 15% of the saving subject to a maximum of Rs. 10,500 if the gross income is above Rs. 1,50,000 and up to Rs. 5,00,000.

(d) Educational Cess 2% of income tax due

(e) Special Rebate: For females, maximum Rs. 5,000 over and above the rebate on savings.

Ans.
1. Find the annual Income.
2. Subtract the total amount of donation from the annual Income.
3. Subtract the standard deduction from the balance income after subtracting the donation.
4. Find the total saving and rebate from saving.

5. Find total tax on her taxable income.
6. Subtract the rebate from saving from calculated I. Tax.
7. Subtract women's rebate of I.Tax from the balance tax.
8. subtract the I.Tax paid by her in last 11months.
9. Before this calculate Education less on payable Income Tax and add it in payable tax.
10. Write the Answer after subtracting the tax paid by her in last 11 months of the financial year.
11. Same as Q.No. 25 of set 1. Anser Rs. 1014/-

Q.24. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of Rs. 80 per sq. m.

OR

The radius of circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area.

Ans.
1. Draw the neat and correct figure of text.
2. Write the formulae of curved surface area of cylinder and cone. ($2\pi rh + \pi rl$)
3. Find the slant height of the cone $l=\sqrt{h^2+r^2}$
4. Find the sum of curved surface area of cylinder & lone.
5. Find the rate of cloth required for tent = rate per sq.m. × Curved surface area of tent.
6. Same as Q.No. 23 of set 1
7. Answer Rs.82720

OR PART

1. Draw the neat and correct figure of frustum.
2. Write the formulae for Total surface area of frustum = $\pi(R+r) + \pi r^2 + \pi r^2$
   $= \pi\{(R+r) + R^2 + r^2\}$
   Now put the values of π, R and r and find the answer.
3. Same as Q No. 23 (or part) of Set 1.
4. Answer 7599.4 Sq. cm.

Q. 25. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

Or

From a building 60 metres high the angle of deprt1ssion of the top and bottom of lamppost are 30° and 60° respectively. Find the distance between lamppost and building. Also find the difference of height between building and lamppost.

Ans.
1. Draw Correct and neat figure according to the question.
2. Take the trigonometric ratio ($\tan 30^\circ$ $\tan 60^\circ$) and solve for x & h.
   Where x is the height of tower and h is the height of flag pole.
3. See Question No.-22 of Set 1.
4. Answer Height of tower - 5.2 mtr, Height of flag pole = 10.4 mtrs.

OR PART

1. Draw correct and neat figure according to the question.
2. Let x be the distance between the building and the lam post and h be the height of the lamp post
3. Now taking trigonometric Ratio we will find out the values of x & h hence the distance between building & lamp post and height of the lamp post.
4. See question No. 22 (or part) of Set 1.
5. Answer. Height of lamp post = 40 mtrs.
   Difference between height of building & that of lamp post = 60- 40 = 20 mtrs.

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